Neutrino mixings from a U(2) flavour symmetry[*]

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Abstract

We extend a previously developed description of the flavour parameters in the charged fermion sector, based on a U(2) flavour symmetry, to include two main features of the neutrino sector seemingly implied by recent data: a large mixing angle $\theta_{\mu\tau}$ and a large hierarchy in the neutrino squared mass differences. A unified description of quark and lepton masses and mixings emerges. The neatest quantitative predictions are for elements of the unitary mixing matrix in the lepton sector:

$$\left| V_{\mu 1}^{\ell} \right| = \left| \frac{V_{e3}^{\ell}}{V_{\mu 3}^{\ell}} \right| = \left| \frac{V_{e2}^{\ell}}{V_{\tau 3}^{\ell}} \right| = \sqrt{\frac{m_e}{m_\mu}},$$

which go together with the analogous relations in the quark sector:

$$\left| \frac{V_{ub}^{q}}{V_{cb}^{q}} \right| = \sqrt{\frac{m_u}{m_c}}, \quad \left| \frac{V_{td}^{q}}{V_{ts}^{q}} \right| = \sqrt{\frac{m_d}{m_s}}.$$

1 Introduction

If neutrinos indeed oscillate, as seemingly implied, at different level of evidence, by several experiments, the number of flavour parameters in the current description of particle physics increases from 13 to 22 in the case of three light Majorana neutrinos: 3 neutrino masses, m_i, and a unitary mixing matrix in the charged leptonic weak current with 3 angles and 3 physical phases. Is there an overall rationale behind these parameters within the current paradigm of particle physics?

Unlike the case of the quark sector, where most of the parameters are known, sometimes even with significant precision, the available information in the neutrino sector is still very scanty. The interpretation of the atmospheric neutrino anomaly in terms of neutrino oscillations requires a large mixing angle θ_{23} between (μ, τ) and two of the neutrino mass eigenstates ν_2, ν_3 [1]. Furthermore the squared mass difference between these two states, Δm^2_{23}, is in the $10^{-2} \div 10^{-3}$ eV^2 range and, likely, significantly larger than the other independent neutrino mass squared difference, $\Delta m^2_{23} \gg \Delta m^2_{12}$, as suggested by solar neutrino experiments [2]. Not much else is reliably known at present, except that no simultaneous explanation is possible, in a three neutrino oscillation picture, of the LSND result [3] together with the atmospheric and solar neutrino anomalies, even if the energy dependence of the suppression of the solar neutrino flux is neglected [4].

In spite of this scanty information several attempts have been made to explain the largeness of θ_{23} in terms of flavour symmetries. This becomes non trivial if one wants a parametric rather than accidental explanation of $\theta_{23} = \mathcal{O}(1)$ and, at the same time, of the hierarchy $\Delta m^2_{23} \gg \Delta m^2_{12}$. It is nevertheless possible, using abelian or non abelian symmetries, in a number of different ways. A next natural step, in the same general direction, is to search for a coherent and testable overall description of quark and lepton masses and mixings, which includes these features of neutrino physics.

A prevailing attitude taken in the literature [5, 6] with respect to this problem can be phrased in SU(5) language as follows. Denoting the three families of SU(5) 10-plets and 5-plets by T_i, $\bar{F}_i$, i=1,2,3, respectively, let us assume that the dominant effective Yukawa couplings are:

$$\lambda^T_{ij} T_i T_j H, \qquad \lambda^F_{ij} T_i \bar{F}_j \bar{H}, \qquad \frac{\lambda^N_{ij}}{M} \bar{F}_i H H \bar{F}_j \tag{1}$$

where H, $\bar{H}$ are the usual Higgs 5-plets and M is a heavy scale, possibly the Planck scale. To the extent that this is true, in the basis for $\bar{F}$ and T where λ^T and λ^N are diagonal, writing λ^F in terms of its diagonal form as:

$$\lambda^F = (V^q)^t \, \lambda^F_{\mathrm{diag}} V^\ell, \tag{2}$$

the unitary matrices V^q and V^ℓ represent the mixing matrices in the quark and lepton weak charged currents respectively. Suppose now that a suitable family symmetry, *e.g.*

an abelian U(1), gives λ^F in the 2,3 sector of the form:

$$\frac{\lambda^{F(2,3)}}{\lambda^F_{33}} = \begin{pmatrix} \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) \\ \mathcal{O}(1) & 1 \end{pmatrix}, \tag{3}$$

where ϵ is a small symmetry breaking parameter of order m_s/m_b or m_μ/m_τ. It is evident from (3) that the right diagonalization matrix gives a large mixing angle in the lepton sector together with a small angle, $V_{cb} = \mathcal{O}(m_s/m_b)$, in the quark sector from the left diagonalization matrix. With some care, it is possible to couple this picture, based on an asymmetric Yukawa coupling matrix, with hierarchical neutrino masses and to extend it to the full three families. Several variations of it, not always reducible to SU(5) language and anyhow mostly employing abelian flavour symmetries, can be found in the literature.

At variance with this case, in this paper we explore the possibility that $\theta_{23} = \mathcal{O}(1)$ and $\Delta m^2_{23} \gg \Delta m^2_{12}$ are explained in a context where the charged fermion mass matrices, m^u, m^d, m^e, as the neutrino Dirac mass matrix m_{LR}, do not have off-diagonal elements in the flavour basis which are significantly asymmetric. More specifically, we look for an extension to the neutrinos of the analysis of the charged fermion mass matrices based on a U(2) flavour symmetry [7]. In Section 2 we summarize for easy of the reader the main features of the U(2) analysis of charged fermion masses. In Section 3 we give conditions for incorporating the relations $\theta_{23} = \mathcal{O}(1)$ and $\Delta m^2_{23} \gg \Delta m^2_{12}$ in an extension of U(2) to the neutrino sector and we derive its quantitative consequences. A possible realization of this general pattern is described in Section 4. Conclusions are summarized in Section 5.

2   U(2) and charged fermions masses

The three family multiplets of matter fields ψ_i, i=1,2,3, transform under U(2) as a doublet and a trivial singlet: $\psi_i = \psi_a \oplus \psi_3$, a=1,2. We view each ψ_i as a **16** of SO(10), therefore including a right-handed neutrino. The flavon fields which can couple to the matter bilinears in a U(2)-invariant way, a triplet S^{ab}, a doublet ϕ^a and an antisymmetric singlet A^{ab}, break hierarchically the flavour group as:

$$\text{U}(2) \xrightarrow{\langle S \rangle, \langle \phi \rangle} \text{U}(1) \xrightarrow{\langle A \rangle} \{e\} \tag{4}$$

where U(1) corresponds, in an appropriate basis, to the subgroup of phase rotations of the lightest family and $\{e\}$ is the unity of U(2). More precisely if, in units of a basic scale M,

$$\frac{\|\langle S \rangle\|}{M} \simeq \frac{\|\langle \phi \rangle\|}{M} \simeq \epsilon \gg \frac{\|\langle A \rangle\|}{M} = \epsilon', \tag{5}$$

where ϵ, ϵ' are two small dimensionless parameters, it is possible to show [8], under general conditions, that $\langle S \rangle$ and $\langle \phi \rangle$ are misaligned in U(2) space only by a relative amount of

order ϵ', or in an appropriate basis and up to order one prefactors:

$$\phi \simeq \begin{pmatrix} \epsilon\epsilon' \\ \epsilon \end{pmatrix} \quad S \simeq \begin{pmatrix} \epsilon'^2 & \epsilon\epsilon' \\ \epsilon\epsilon' & \epsilon \end{pmatrix}. \tag{6}$$

Allowing for a general U(2)-invariant Yukawa coupling to Higgs fields containing weak doublets VEVs, this gives the following structure of the Yukawa coupling matrices in flavour space:

$$\frac{\boldsymbol{\lambda}}{\lambda_{33}} = \begin{pmatrix} \epsilon'^2 & \epsilon' & \epsilon\epsilon' \\ \epsilon' & \epsilon & \epsilon \\ \epsilon\epsilon' & \epsilon & 1 \end{pmatrix}. \tag{7}$$

For every entry the corresponding size of the U(2) breaking parameter is indicated. It is only in the case of the 12 and 21 entries that U(2) implies a specific relation, $\lambda_{12} = -\lambda_{21}$, up to correction of relative order ϵ.

Al least to account for $m_c/m_t \ll m_s/m_b \simeq m_\mu/m_\tau$ and $m_u/m_t \ll m_d/m_b \simeq m_e/m_\tau$ a vertical structure has to be supplemented in (7). Taking advantage of the antisymmetry in flavour space of the 12, 21 couplings as opposed to the symmetry of the 11, 22 elements, it is possible to further suppress every entry of the entire 12-block of the λ^u matrix by an SU(5)-breaking parameter ρ [7]. As an example the flavon A^{ab} may be an SO(10) singlet or a **45** of SO(10) with a SU(5) symmetric VEV, whereas S^{ab} can be a **45** of SO(10) with a VEV in the B - L direction. As long as there is no other SU(5) breaking VEV, the 12-block of the λ^u matrix vanishes, since u and u^c belong to the same SU(5) multiplets, unlike the case for d, d^c or e, e^c or ν_L, ν_R. At the same time, up to SU(5) breaking corrections, $\left|\lambda_{21}^d\right| = \left|\lambda_{12}^d\right| = \left|\lambda_{21}^e\right| = \left|\lambda_{12}^e\right|$ and $3\left|\lambda_{22}^d\right| = \left|\lambda_{22}^e\right|$, since d, e^c and d^c, e live in the same SU(5) multiplets, whereas no special relation is implied for the Dirac neutrino mass matrix λ^{LR}, although also non vanishing in the SU(5) limit.

In summary, we are led to the following dependence of the mass matrices on the U(2) and SU(5) symmetry breaking parameters (all normalized to the scale M) ϵ, ϵ' and ρ respectively:

$$\left(\frac{\boldsymbol{\lambda}}{\lambda_{33}}\right)_{d,e,LR} \simeq \begin{pmatrix} \epsilon'^2 & \epsilon' & \epsilon\epsilon' \\ \epsilon' & \epsilon & \epsilon \\ \epsilon\epsilon' & \epsilon & 1 \end{pmatrix}, \tag{8}$$

$$\left(\frac{\boldsymbol{\lambda}}{\lambda_{33}}\right)_u \simeq \begin{pmatrix} \epsilon'^2\rho & \epsilon'\rho & \epsilon\epsilon' \\ \epsilon'\rho & \epsilon\rho & \epsilon \\ \epsilon\epsilon' & \epsilon & 1 \end{pmatrix}, \tag{9}$$

with the particular relations:

$$\lambda_{12}^{u,d,e,LR} = -\lambda_{21}^{u,d,e,LR}, \tag{10}$$

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$$\left|\lambda_{22}^e\right| = 3\left|\lambda_{22}^d\right|, \qquad \left|\lambda_{12}^e\right| = \left|\lambda_{12}^d\right|, \qquad \left|\lambda_{33}^e\right| = \left|\lambda_{33}^d\right|. \tag{11}$$

Allowing for prefactors of order unity in (8) and (9) consistent with (10,11), all presently known properties of quarks and charged leptons are well reproduced by these mass matrices with $\epsilon \simeq \rho \simeq 2 \times 10^{-2}$ and $\epsilon' \simeq 4 \times 10^{-3}$ [7]. Eqs. (11) give, in particular, the well known Georgi-Jarlskog relations among fermion masses [9]. As shown elsewhere [10], the relations [11]:

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{\frac{m_u}{m_c}}, \qquad \left|\frac{V_{td}}{V_{ts}}\right| = \sqrt{\frac{m_d}{m_s}}, \tag{12}$$

implied by (8–10), valid up to corrections of relative order ϵ, represent a test in qualitative agreement with present data, whose significance should improve considerably in the near future.

3   Extension to neutrinos: general considerations

The extension to neutrinos requires knowing the symmetry properties of the right-handed neutrino mass matrix, m_{RR}, entering the see-saw formula for the light neutrinos $m_{LL} = m_{LR} m_{RR}^{-1} m_{LR}^t$. In general m_{RR} arises from $\overline{\mathbf{126}}$ representations of SO(10), fundamental or effective, also transforming under U(2) as singlets, Ω, doublets, Ω^a, or triplets Ω^{ab}. The U(2) antisymmetric singlet does not couple to a neutrino bilinear.

Which structure of m_{RR} could give a large θ_{23} angle and the neutrino mass hierarchy? Note that $\theta_{23} = \mathcal{O}(1)$ should come from the diagonalization of m_{LL} in view of the form (8) of λ^e. Note also that, if all the Ω's were present with the maximal strength consistent with U(2)-breaking, *i.e.* $\|\langle\Omega\rangle\|/M = \mathcal{O}(1)$, $\|\langle\Omega^a\rangle\|/M \simeq \|\langle\Omega^{ab}\rangle\|/M = \mathcal{O}(\epsilon)$, the resulting m_{LL} would not have any of the desired properties.

In view of this, based on a classification of the possible forms of m_{LL} giving $\theta_{23} = \mathcal{O}(1)$ and $\Delta m_{23}^2 \gg \Delta m_{12}^2$ [6], we consider the following ansatz for the Ω's: i) The singlet Ω should be absent or sufficiently suppressed; ii) $\|\langle\Omega^a\rangle\| \simeq \|\langle\Omega^{ab}\rangle\| \simeq \epsilon M$ and, in the U(2) basis where $\Omega^1 = 0$,

$$\frac{\Omega^{ab}}{M} \simeq \begin{pmatrix} 0 & \epsilon\epsilon' \\ \epsilon\epsilon' & \epsilon \end{pmatrix}. \tag{13}$$

Again $\Omega^{11} = 0$ means that Ω^{11} is sufficiently suppressed. As we shall see in the next Section, all this can be explicitly implemented in at least one concrete example.

We argue that the resulting m_{RR} gives the desired properties of m_{LL} under a suitable condition. It is, in the same flavour basis as (8),

$$m_{RR} = M \begin{pmatrix} 0 & \epsilon\epsilon' & 0 \\ \epsilon\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix}, \tag{14}$$

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which has, in this approximation, a massless eigenstate:

$$N_l \simeq N_1 + \mathcal{O}(\epsilon') N_3 \tag{15}$$

and two heavy states, N_h^1 and N_h^2, of similar masses M_{h_1}, M_{h_2}, predominantly composed of N_2 and N_3 with comparable coefficients. If expressed in the basis of these right-handed neutrinos mass eigenstates, $N_R = (N_l, N_h^1, N_h^2)$, the Dirac mass matrix m_{LR} of (8) acquires the form:

$$\frac{m'_{LR}}{m'^{33}_{LR}} \simeq \begin{pmatrix} \epsilon'^2 & \epsilon' & \epsilon' \\ \epsilon' & \epsilon & \epsilon \\ \epsilon' & 1 & 1 \end{pmatrix}. \tag{16}$$

Notice that we have achieved a lighter right-handed neutrino N_l, massless in the exact limit of (14), predominantly coupled to ν_μ and ν_τ with comparable strength. This is the key for having a large θ_{23} angle and, at the same time, hierarchical left-handed neutrinos [12]. If N_l is light enough, $M_l \ll M_h$, the dominant terms from N_R exchanges in the mass Lagrangian of the left-handed neutrinos are:

$$\mathcal{L}_{m_{LL}} \simeq \frac{v^2}{M_l} \epsilon'^2 \left(\nu_\mu + \nu_\tau + \epsilon' \nu_e \right)^2 + \frac{v^2}{M_h} \nu_\tau^2 \tag{17}$$

where we have set $m_{LR}^{33} \simeq v$, a typical SU(2) $\times$ U(1) breaking vacuum expectation value, and all terms are meant to have a numerical coefficient of order unity. Therefore, if

$$\frac{M_l}{M_h} \ll \epsilon'^2, \tag{18}$$

the left-handed neutrino masses are hierarchical:

$$m_3 \simeq \frac{v^2 \epsilon'^2}{M_l} \gg m_2 \simeq \frac{v^2}{M_h} \tag{19}$$

and m_{LL} is diagonalized by an order 1 rotation in the ν_μ/ν_τ - 2/3 sector, up to further rotations with angles of order ϵ'.

The fact that $\mathcal{L}_{m_{LL}}$ in (17) is diagonalized, to a good approximation, by an order 1 rotation in the ν_μ/ν_τ sector:

$$V^\nu \cong \begin{pmatrix} 1 & \mathcal{O}(\epsilon') & \mathcal{O}(\epsilon') \\ \mathcal{O}(\epsilon') & \cos\bar{\theta} & \sin\bar{\theta} \\ \mathcal{O}(\epsilon') & -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix}, \tag{20}$$

together with the explicit form of λ^e in eqs. (8) and (10), has an important implication. The mixing matrix in the leptonic charged weak current $\bar{\ell}\gamma_\mu V^\ell \nu$ is in fact given by:

$$V^\ell = (V^E)^\dagger V^\nu, \tag{21}$$

where V^E is the left unitary rotation needed to diagonalize λ^e. In view of (8,10) it is:

$$V^E \cong \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \mathcal{O}(\epsilon) \\ 0 & \mathcal{O}(\epsilon) & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_E & \sin\theta_E & 0 \\ -\sin\theta_E & \cos\theta_E & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{22}$$

where $\tan\theta_E = \sqrt{(m_e/m_\mu)}$. Therefore we obtain a mixing matrix:

$$V^\ell \cong \begin{pmatrix} \cos\theta_E & \sin\theta_E\cos\theta & \sin\theta_E\sin\theta \\ -\sin\theta_E & \cos\theta_E\cos\theta & \cos\theta_E\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \tag{23}$$

or, adopting the common notation for V^ℓ in terms of 2×2 rotations,

$$V^\ell = R_{23}(\theta_{23})R_{13}(\theta_{13})R_{12}(\theta_{12}), \tag{24}$$

one has

$$\theta_{12} = \sqrt{\frac{m_e}{m_\mu}}\cos\theta_{23}, \qquad \theta_{13} = \sqrt{\frac{m_e}{m_\mu}}\sin\theta_{23}. \tag{25}$$

These relations between θ_{12}, θ_{13} and θ_{23}, valid up to correction of relative order ϵ, are the analogue in the lepton sector of eqs. (12). For appropriate values of the neutrino masses, as discussed below, they are consistent with a neutrino oscillation interpretation of the atmospheric and solar neutrino anomalies, as known today. In particular, from $\sin^2 2\theta_{23} > 0.9$ we obtain

$$\sin^2 2\theta_{12} = (6 \div 13) \times 10^{-3}, \tag{26}$$

well compatible with the small angle MSW interpretation of the solar neutrino data[1] [2].

4   An explicit example

In this Section we briefly discuss an explicit realization of the picture described previously. Other than showing a concrete example, there are two related reasons for doing this. The zeros in (14) will in general be replaced by small but non-vanishing entries. In turn these entries determine if condition (18) is satisfied and, at the same time, fix the order of magnitude of the neutrino masses via (19). Since condition (18) requires:

$$\det m_{RR} = M_h^1 M_h^2 M_l \ll \epsilon^3 \epsilon'^2 M^3, \tag{27}$$

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[1]The second of (25) implies $\theta_{13} = (2 \div 3)^\circ$ compatible with the CHOOZ limit, $\theta_{13} < 13^\circ$ if $\Delta m^2_{\text{atm}} > 2 \times 10^{-3}$ eV^2 [13], and with the somewhat less stringent limits from Super-Kamiokande: $\theta_{13} \lesssim 20^\circ$ [14].

barring cancellations among terms of the same order of magnitude, it must be:

$$m_{RR}^{33} \ll M\epsilon, \qquad m_{RR}^{11} \ll M\epsilon\epsilon'^2, \qquad m_{RR}^{13} \ll M\epsilon\epsilon'. \qquad (28)$$

A possible concrete realization is obtained by assuming that Ω^a and Ω^{ab} are both fundamental fields, transforming as $\overline{\mathbf{126}}$ of SO(10), to be added to the flavons ϕ^a, S^{ab} and A^{ab} introduced in Section 2, that couple to charged fermions bilinears. In close analogy with the discussion made in ref. [8], by considering an SO(10) × U(2) invariant potential which also includes all flavon fields with opposite SO(10) × U(2) transformation properties, denoted by a bar, it is possible to show that the appropriate SU(3) × SU(2) × U(1) invariant components of Ω^a, Ω^{ab}, ϕ^a are aligned in U(2) space as:

$$\Omega^a \simeq \phi^a, \qquad \Omega^{ab} \simeq \frac{1}{M\epsilon}\left(\Omega^a \phi^b + \Omega^b \phi^a\right), \qquad (29)$$

to a sufficiently good approximation, so that (28) are fulfilled[2]. More precisely in the U(2) basis where $\Omega^a = M(0, \epsilon)$, eqs. (6,13) hold and the dominant correction to (14) arise from the couplings:

$$\frac{1}{M}\nu_{R3}(\Omega^a\bar{\phi}_a + \Omega^{ab}\bar{S}_{ab})\nu_{R3},$$

$$\frac{1}{M}\nu_{Ra}(\Omega^{ab}\bar{\phi}_b + \frac{1}{M}A^{ab}\Omega^c\bar{S}_{bc})\nu_{R3}, \qquad (30)$$

$$\frac{1}{M^2}\nu_{Ra}(S^{ab}\Omega^c\bar{\phi}_c + A^{ac}\Omega^{bd}\bar{S}_{cd})\nu_{Rb},$$

which give respectively:

$$m_{RR}^{33} \simeq M\epsilon^2 \quad m_{RR}^{13} \simeq M\epsilon^2\epsilon' \quad m_{RR}^{11} \simeq M\epsilon^2\epsilon'^2 \ . \qquad (31)$$

One has therefore:

$$m_{RR} \simeq M\epsilon \begin{pmatrix} \epsilon\epsilon'^2 & \epsilon' & \epsilon\epsilon' \\ \epsilon' & 1 & 1 \\ \epsilon\epsilon' & 1 & \epsilon \end{pmatrix} \qquad (32)$$

which gives $M_h^1 \simeq M_h^2 \simeq \epsilon M$ and $M_l \simeq \epsilon^2\epsilon'^2 M$ or, from eqs. (19):

$$m_3 \simeq \frac{v^2}{M\epsilon^2}, \qquad \frac{m_2}{m_3} \simeq \epsilon. \qquad (33)$$

Taking $v = 250\,\text{GeV}$, $M = M_{\text{Planck}}$ and $\epsilon = 2 \times 10^{-2}$ as required to describe the quark parameters [7], we obtain

$$\Delta m_{23}^2 \simeq m_3^2 = \mathcal{O}(10^{-2}\,\text{eV}^2)$$

$$\Delta m_{12}^2 \simeq m_2^2 = \mathcal{O}(10^{-5} \div 10^{-6}\,\text{eV}^2)$$

(34)

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[2]The same approximate alignment holds for the barred fields.

which, together with (25), can give a consistent description of atmospheric and solar neutrino data so far. Note that the lightest right-handed neutrino has a mass of order 10^{10} GeV.

Before closing this Section, we would like to comment on the possibility of accommodating in a U(2) model alternative patterns of neutrino masses and mixings than the one described so far, always accounting in a parametric way for $\theta_{23} = \mathcal{O}(1)$ and $\Delta m_{23}^2 \gg \Delta m_{12}^2$[3]. This is a difficult question to answer in general. We have been able to find, however, an alternative example always based on the dominance of a light right-handed neutrino, coupled with comparable strength to ν_μ and ν_τ. In this model we have, besides the ϕ, S, A flavons, a fundamental $\overline{\mathbf{126}}$, Ω_A^{ab}, which is an antisymmetric tensor under U(2) and has a vacuum expectation value $|\Omega_A| \simeq \epsilon' M$. In this case the dominant operators contributing to the different entries of m_{RR} are as follows:

$$m_{RR} \simeq \Omega_A^{12} \begin{pmatrix} \bar{S}_{22}A^{12} & \bar{S}_{22}S^{22} + \bar{\phi}_2\phi^2 & \bar{\phi}_2 \\ \bar{S}_{22}S^{22} + \bar{\phi}_2\phi^2 & \bar{A}_{12}S^{22} & \bar{\phi}_1 + \bar{A}_{12}\phi^2 \\ \bar{\phi}_2 & \bar{\phi}_1 + \bar{A}_{12}\phi^2 & \bar{A}_{12} \end{pmatrix} \tag{35}$$

so that:

$$m_{RR} \simeq M\epsilon\epsilon' \begin{pmatrix} \epsilon' & \epsilon & 1 \\ \epsilon & \epsilon' & \epsilon' \\ 1 & \epsilon' & \epsilon'/\epsilon \end{pmatrix}. \tag{36}$$

When inserted in $m_{LL} = m_{LR}m_{RR}^{-1}m_{LR}^t$ together with (8) this gives:

$$\theta_{23} \simeq 1, \qquad \theta_{12} \simeq \theta_{13} \simeq \frac{\epsilon'}{\epsilon} \tag{37}$$

and

$$m_3 \simeq \frac{v^2\epsilon}{M\epsilon'^2}, \qquad \frac{m_2}{m_3} \simeq \left(\frac{\epsilon'}{\epsilon}\right)^2. \tag{38}$$

Even though we loose the exact predictions (25), a qualitative description of the data may be possible also in this case, with appropriate $\mathcal{O}(1)$ prefactors.

5 Conclusions

In this paper we have attempted a unified description of quark and lepton masses and mixings, based on a U(2) flavour symmetry. More precisely, we have extended a previously developed description of the flavour parameters of the charged fermions to include two main features of the neutrino sector seemingly implied by recent experimental findings: a

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[3]Alternative U(2) models not trying to incorporate parametrically $\theta_{23} = \mathcal{O}(1)$ and $\Delta m_{23}^2 \gg \Delta m_{12}^2$ in a 3 neutrino scheme can be found in refs. [15].

large mixing angle between ν_μ and ν_τ and a large hierarchy in the neutrino squared mass differences, relevant to the oscillation phenomena. We have shown that this is possible using an interplay between the flavour U(2) symmetry and the vertical SO(10) symmetry. All the qualitative features of the spectra and mixings of quarks and leptons can be accounted for by two small parameters $\rho \simeq \epsilon$ and ϵ' expressing respectively the breaking scales of SO(10) $\times$ U(2) to SU$_{3,2,1} \times$ U(1) and to SU$_{3,2,1}$ relative to a basic scale M, close to the Planck mass.

From a phenomenological point of view, the neatest quantitative predictions in the neutrino sector are given in eqs. (25) or, independently from the chosen parametrization of the mixing matrix in the lepton sector,

$$\left| V_{\mu 1}^\ell \right| = \left| \frac{V_{e3}^\ell}{V_{\mu 3}^\ell} \right| = \left| \frac{V_{e2}^\ell}{V_{\tau 3}^\ell} \right| = \sqrt{\frac{m_e}{m_\mu}}, \tag{39}$$

with a negligibly small CP-phase. These relations, similarly to the analogous relations in the quark sector, eqs. (12), should be valid to a good approximation and should allow a quantitative test of the overall picture. In a specific realization, we find also the order of magnitude of the neutrino squared mass differences given in eqs. (34). All of these relations are compatible with the present experimental information both in the quark sector and in the lepton sector. Precise comparisons should be possible in a not too distant future.

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